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# N-Particle Cross Sections in Diffraction Production Models

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ABSTRACT

We show how a simple $t_{min}$ effect can significantly alter the $\sigma_n \sim 1/n^2$ rule found in most diffractive models of particle production. Data on pp, $\pi^-p$, and $K^-p$ collisions appears consistent with $n^2\sigma_n \sim \exp -n^4/s$, which implies: (1) double diffraction dissociation is the dominant feature, (2) $\langle n\rangle \sim \log s$, but correlation functions rise much more slowly than before. Critical comments on diffractive models are given.

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In this short note we would like to comment on the relationship between the observed cross sections, $\sigma_n(s)$, to produce n charged particles at incident energy $s \approx 2\, p_{lab}$ and the variety of diffractive production[1] or nova[2] models. These models have as one of their simple common features a large n behavior of $\sigma_n(s)$ at fixed s which is

$$\sigma_n(s) \underset{\text{fixed } s}{\sim} 1/n^2 . \tag{1}$$

This is built in so that the average multiplicity

$$\langle n \rangle = \sum^{\sqrt{s}} n\, \sigma_n \sim \log s . \tag{2}$$

Naively, the experimental results[3] for $\sigma_n$ at $p_{lab}$ = 50, 69, 100, 200, and 300 GeV/c incident momentum for charged particles produced in pp collisions would appear to considerably reduce the credibility of such models. In Fig. 1 we have plotted the observed $n^2\, \sigma_n$ versus $n^4$ at each of the mentioned energies. If $\sigma_n \sim 1/n^2$, one should see a horizontal line; one does not.

This effect is not restricted to charged particle distributions from pp collisions. In Fig. 2 we show[4] $\sigma_n$ for $\pi^- p$ collisions at $\pi^-$ lab momentum of 50 GeV/c and $\sigma_n$ for $K^- p$ interactions at 33.8 GeV/c. Each of these is presented as $n^2 \sigma_n$ plotted against $n^4$; once again exponential deviations from the simplest expectation is observed.

As has also been noted by Hwa,[5] there is at least one piece of the physics, even within the diffractive models, which has been omitted from a straightforward $\sigma_n \sim 1/n^2$. Namely, when one produces fast forward or backward fireballs, as is supposed in such models, the strong damping of the production amplitude in the momentum transferred from the beam or target to the fireball will lead to a strong suppression of large n events because at finite energies the minimum momentum transfer, $t_{min}$, is not negligible.

We will explore in this note the consequences of the most simple implementation of this effect; namely, we just modify the production amplitude by multiplying it by $\exp - b\,|t_{min}|$. Hwa is more sophisticated; the physics is the same.

We wish to make the following points:

(1) At a single energy this simple incorporation of the $t_{min}$ effect does indeed yield a reasonable description of the data.

(2) The same model seems to work at 50, 69, 100, 200 and 300 GeV/c only if the diffractive production mechanism is <u>double</u> fireball. Previous analyses[6] of inclusive data in diffraction models have used approximately a mixture of 2/3 single dissociation and 1/3 double. It appears to us that any <u>diffractive</u> model which can preserve the nice agreement with data found in previous work and describe the $\sigma_n$ data now available must be so different from existing versions that one is back at the beginning.

(3) In our elementary modification of $\sigma_n$ where we tack on a factor $\exp(-b|t_{min}|) \approx \exp -n^4/s$, one still has $\langle n \rangle \sim \log s$, but all higher moments are altered. In particular, the energy dependence of $\langle n^2 \rangle$ or $f_2 \equiv \langle n(n-1) \rangle - \langle n \rangle^2$ is $s^{1/4}$ instead of the popular $s^{1/2}$.

(4) In this altered diffractive model, the log s behavior of $\langle n \rangle$ arises from the <u>small</u> n aspect of $\sigma_n$. This is both unsatisfying and rather unlike what the data would seem to be telling us.

(5) If one is willing to further alter the rules, one can describe the prong distribution for $n \geq 10$ from 50 to 300 GeV/c by

$$n^2 \sigma_n = 137\, s^{1/4} \exp - (n^4/175 s), \tag{3}$$

s in $(\mathrm{GeV})^2$. We have no explanation for this curiousity. {It implies for example that $\langle n \rangle \sim s^{1/4} \log s$ and $\langle n^2 \rangle \sim s^{1/2}$.} However, we offer it up in that spirit.

(6) In spite of the fact that the pp, $\pi^- p$, and $K^- p$, $\sigma_n$ data are described by $n^2 \sigma_n \sim \exp -n^4/s$, we suspect that diffractive production is not a sizeable part of $\sigma_{total}$ at any energy. The simple numerical reason for this is that even this rather adequate parametrization by no means describes the peak in $\sigma_n$ where most events occur. Certainly physical mechanisms other than those at the heart of diffractive models are responsible for the bulk of the data.

## MODEL

The details of the production mechanism are normally not specified in diffractive models, so as mentioned we shall choose to modify the previous predictions by the simplest physically reasonable possibility: the fireballs are produced with a distribution proportional to $\exp - b|t|$. No doubt one may invent and explore the consequences of more elaborate parametrizations of the t-distribution (exponentials multiplied by Bessel functions come to one's mind) to incorporate the damping need to suppress large n in $\sigma_n$.[7] Leaving this to the reader, we elaborate on our heuristic model.

For single fireball production pp → p + missing mass M one has for large s

$$-t_{min} \approx M^4 m_p^2 / s^2 , \tag{4}$$

while for double production pp → M + M′ one has

$$-t_{min} \approx M^2 M'^2 / s . \tag{5}$$

We choose $M \approx M'$ and take M proportional to the number of produced particles

$$M = cn , \tag{6}$$

where[8] $c \approx 300$ MeV. We then expect

$$n^2\sigma_n \approx \exp - \alpha n^4/s^2 \quad (\text{single fireball}), \tag{7}$$

$$n^2\sigma_n \approx \exp - \beta n^4/s \quad (\text{double fireball}), \tag{8}$$

Fig. 1 shows that the data for $n^2\sigma_n$ indeed do fall exponentially in $n^4$ over a range of energies.

However, as $p_{lab}$ varies from 50 to 200 GeV/c one has s varying by a factor of 4 and $s^2$ by a factor of 16. If the $t_{min}$ effect can "save" the diffractive models one surely wants essentially the same slope $\alpha$ or $\beta$ to occur above.

In fact, the slopes at 50, 69, 100, 200, and 300 GeV/c are remarkably consistent with the 1/s of the double fireball; if there is any $1/s^2$ variation at 50 GeV/c it would give a much flatter and dominant contribution at, say, 200 GeV/c.

As a consistency check we note that for c ranging from 200-300 MeV/c we find the slope b ranging from 3.5 - 0.7 $\text{GeV}^{-2}$, certainly a reasonable sort of value.

On the other hand, previous analysis using diffractive models gave large single firaball contributions -- then the $1/s^2$ means $t_{min}$ has very little effect and cannot "save" the models. It thus appears that there is real contradiction between the $\sigma_n$ vs. n behavior and previous results for diffractive models; it would appear that the diffractive cross section cannot make up most of $\sigma_T$, except possibly if considerable refinements are introduced into the models.

Once one takes a form for $\sigma_n$ of the type

$$n^2 \sigma_n \sim \exp - a n^p / s^q , \tag{9}$$

where p and q are some numbers, he finds that the moments

$$\langle n^\ell \rangle = \sum^{\sqrt{s}} n^\ell \sigma_n . \tag{10}$$

are altered from the kindergarten $\sigma_n \approx 1/n^2$ model for $\ell > 1$. Elementary estimates of $<n^\ell>$ using (9) yield

$$\langle n \rangle \sim \frac{q}{p} \log s + \text{constant} + O(1/s^q) , \tag{11}$$

as before, but

$$\langle n^\ell \rangle \sim \frac{1}{p} \Gamma\left(\frac{\ell-1}{p}\right) \left(s^{q/p}\right)^{\ell-1} + \text{constant}, \ \ell > 1. \tag{12}$$

For the double diffraction dissociation which the data seems to be choosing, $q = 1$ and $p = 4$. So

$$\langle n \rangle \sim \log s \tag{13}$$

but

$$\langle n^2 \rangle \sim s^{1/4} \tag{14}$$

which differs markedly from the $s^{1/2}$ one usually predicts in the diffractive models. Of course additional energy dependence for $<n^\ell>$ can result if $\sigma_n$ is endowed with an energy dependent coefficient, as in Eq. 3 where $<n> \sim s^{1/4} \log s$, $<n^2> \sim s^{1/2}$, etc.

The reason one continues to find $\langle n \rangle \sim \log s$ is that this behavior comes from the small n dependence of $\sigma_n$, while higher moments probe the large n behavior of $\sigma_n$, which we have radically altered. Now, even preserving this attractive feature of $\langle n \rangle \sim \log s$ is, from the point of view of physics, not to be regarded as an achievement. This is because "small" n for any formula like (9) must still be larger than the value where $\sigma_n$ has its maximum and typically seems to be $n \approx 10$ or so at the energies we considered. Since, for n this large, $\sigma_n$ is already considerably smaller than at its maximum, the net log s multiplicity can only be receiving a moderately small contribution, at present machine energies, from diffractive mechanisms. There would appear to be some physics of the production process which is not accounted for in the kind of straightforward diffraction models as we have interpreted them here.

Berger[9] has taken a different point of view but arrives at conclusions similar to ours if we understand him correctly. He preserves the $1/n^2$ behavior with no $e^{at}$ modification. But he makes the parameters of the model energy dependent so that it is no longer a diffractive model (i.e., no longer an energy independent production mechanism). We agree that a diffractive model is probably useful only for a small part of $\sigma_T$. He still feels, however, that the model with energy dependent parameters is a useful way to describe data; it will be interesting to see what is required to simultaneously describe at NAL energies the $\sigma_n$ data vs. hard s, and the inclusive spectra which exhibit limiting behavior. In addition, Berger appears to have a remarkable prediction in his analysis. He has the total number of pions falling as $1/n^2$, while the number of charged pions falls as $1/n!$ Thus for large n he expects essentially only $\pi^0$ production, $n_\pi \approx n_{\pi^0}$. This would appear to be inconsistent with the ISR photon data, but perhaps it needs to be tested more directly.

For completeness, if not best-fit-ness, we present in Fig. 1 an adequate parametrization of the 50 - 200 GeV/c pp data which does have $\langle n \rangle \sim \log s$:

$$n^2 \sigma_n = 700 \text{ mb} \exp -(70/\ell)(n/10)^4 . \tag{15}$$

We may also describe the $\pi^- p$ or $K^- p$ data in Fig. 2 by such a formula, but have no right to take the s dependence seriously since we have given $\sigma_n$ at only one s for each process. However, if we assume that the same physics is operative then, for example, we suggest for $\pi^- p \to n$ charged particles the formula

$$n^2 \sigma_n (\pi^- p) = 460 \text{ mb} \exp -(98/\ell)(n/10)^4 , \tag{16}$$

which is a fit at $s = 100 \, (\text{GeV})^2$ for $n \geq 8$ and a prediction to be tested at Mirabelle and NAL for larger s values.

Finally, independent of considerations concerning models, we note that the $\sigma_n$ given in Eq. 3 for pp collisions quite adequately describes all of the data from 50 to 300 GeV/c as shown in Fig. 3. This is a useful and interesting result, but its interpretation is not clear (to us). Whatever the detailed mechanism one is tempted to take seriously the connection of $e^{-n^4/s}$ with a $t_{min}$ effect. The form of Eq. (3) is of course not unique; it will be interesting to see whether it will remain correct in the fact of increasingly accurate data over a large energy range.

## REFERENCES

[1] R.C. Hwa, Phys. Rev. Letters, 26, 1143 (1971) and references therein.

[2] M. Jacob and R. Slansky, Phys. Rev. D5, 1847 (1972).

[3] 50 GeV/c and 69 GeV/c: Soviet-French Collaboration, paper submitted to the XVIth International Conference on High Energy Physics, Sept., 1972. 100 GeV/c: Michigan-Rochester Collaboration, paper submitted to the XVIth International Conference on High Energy Physics, Sept. 1972. 200 GeV/c: G. Charlton, et al., Phys. Rev. Letters 29, 515 (1972). 300 GeV/c: F. T. Dao, et al., NAL-UCLA Collaboration, paper submitted to the XVIth International Conference on High Energy Physics, Sept., 1972.

[4] Aachen-Berlin-CERN-IHEP-London (I. C.)-Mons-Saclay-Vienna Collaboration, paper submitted to the XVIth International Conference on High Energy Physics, Sept., 1972.

[5] R.C. Hwa, Univ. of Oregon preprint, August, 1972.

[6] Ref. 1 and 2.

[7] We thank H. Navlet for informative discussions on this point.

[8] J. Van der Velde, private communication.

[9] E. L. Berger, paper submitted to the XVIth International Conference on High Energy Physics, Sept., 1972.

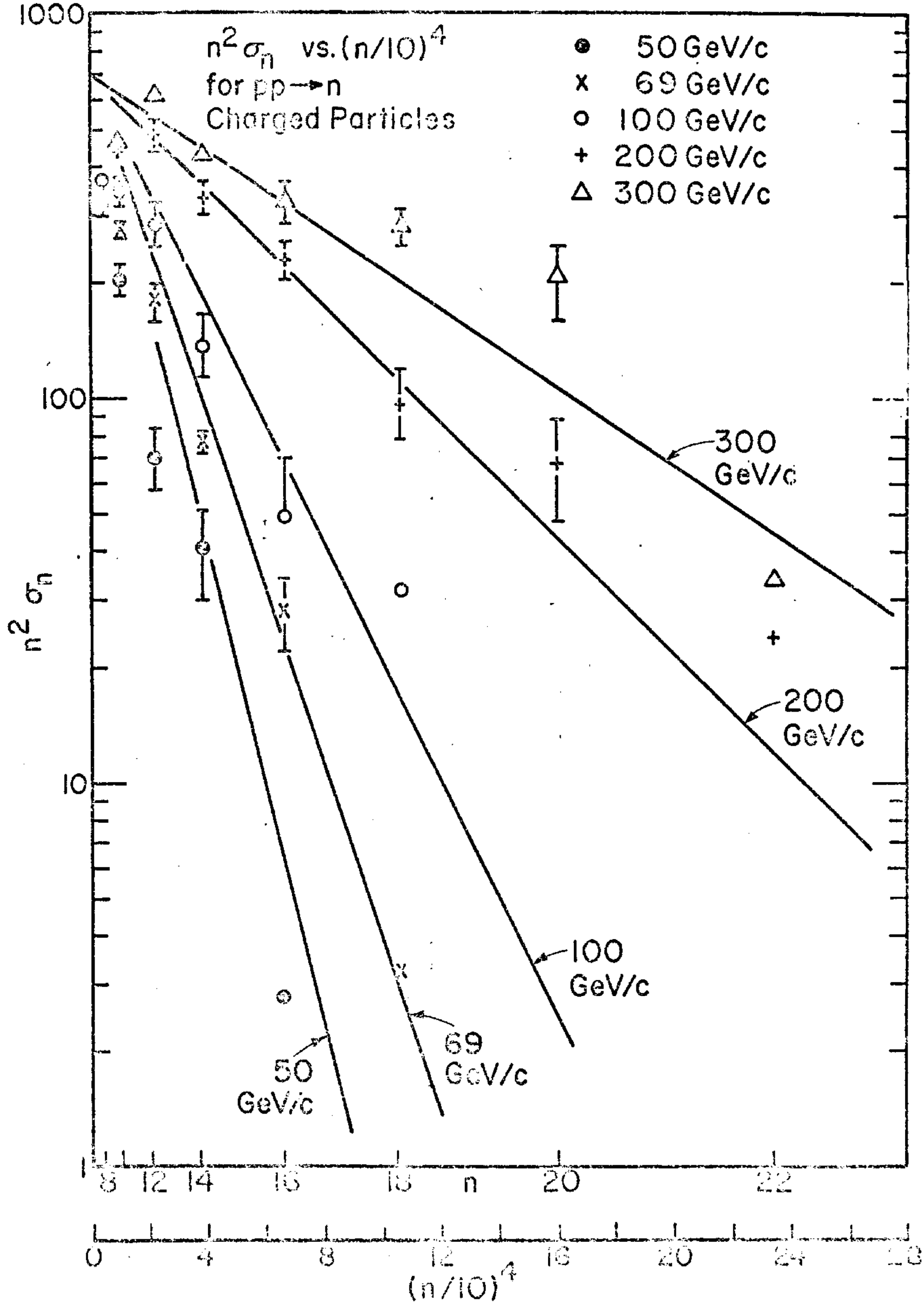
## FIGURE CAPTIONS

Figure 1: The data from Ref. 3 on $\sigma_n$ from pp collisions at 50, 69, 100, 200, and 300 GeV/c incident lab momentum. Representative error bars are shown. The lines imposed on the data are $n^2\sigma_n$ = 700 mb exp $-(70/s)(n/10)^4$, with s in $(\mathrm{GeV})^2$. This is a modification of the simple $1/n^2$ law.which may indicate significant <u>double dissociation</u> in a diffractive picture.

Figure 2: The data from Ref. 4 for $\sigma_n$ from $\pi^- p$ collisions at 50 GeV/c and $K^- p$ at 33.8 GeV/c. The line through the $\pi^- p$ data indicates a $n^2\sigma_n = 460\,\mathrm{mb}\ \exp - (98/s)(n/10)^4$ rule which may be tested at larger s.

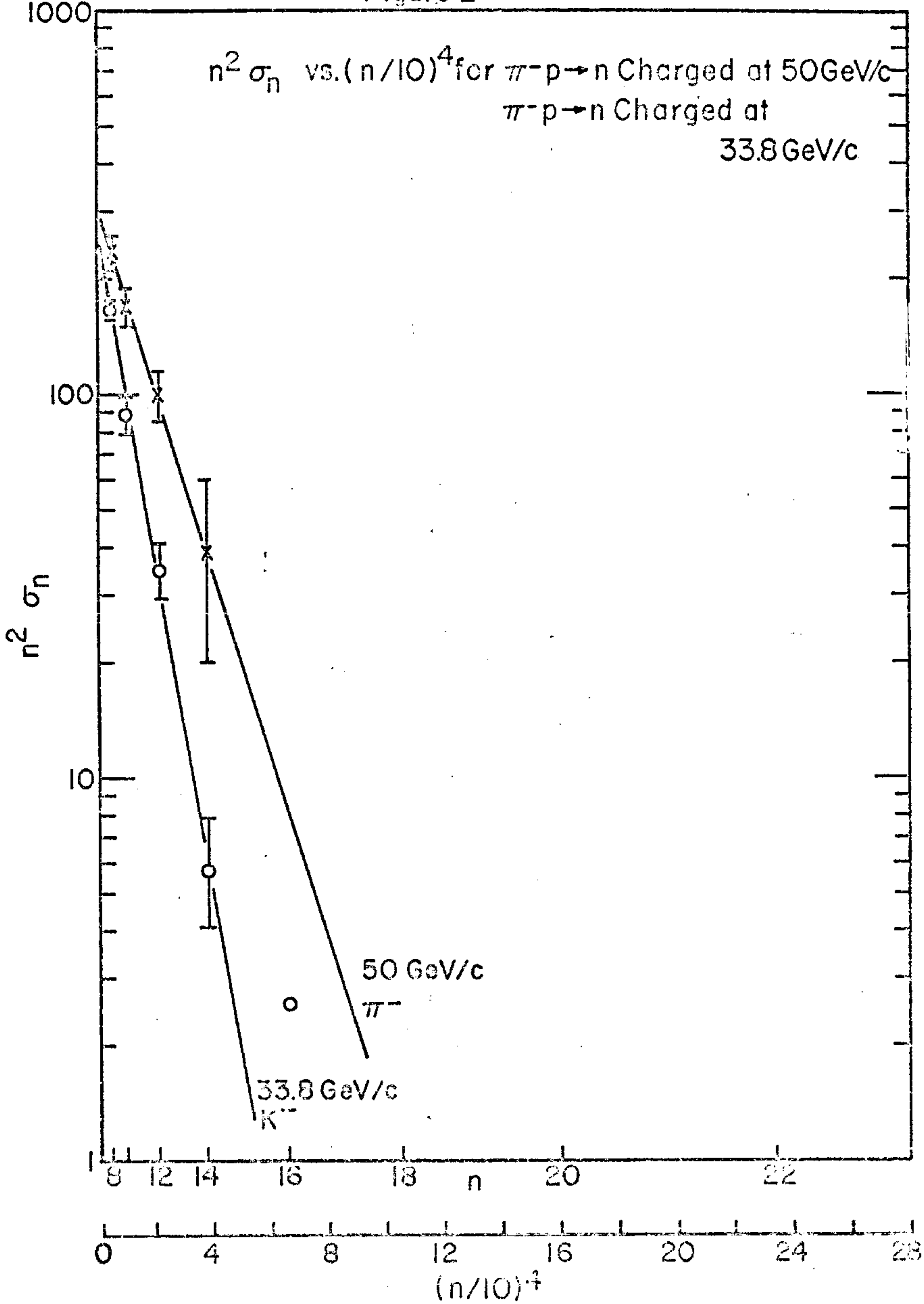
Figure 3: The pp data from 50 to 300 GeV/c are shown again along with the lines predicted by $n^2\sigma_n = 137(s)^{1/4}$ mb exp $- n^4/175s$, s in $(\mathrm{GeV})^2$.

Figure 1



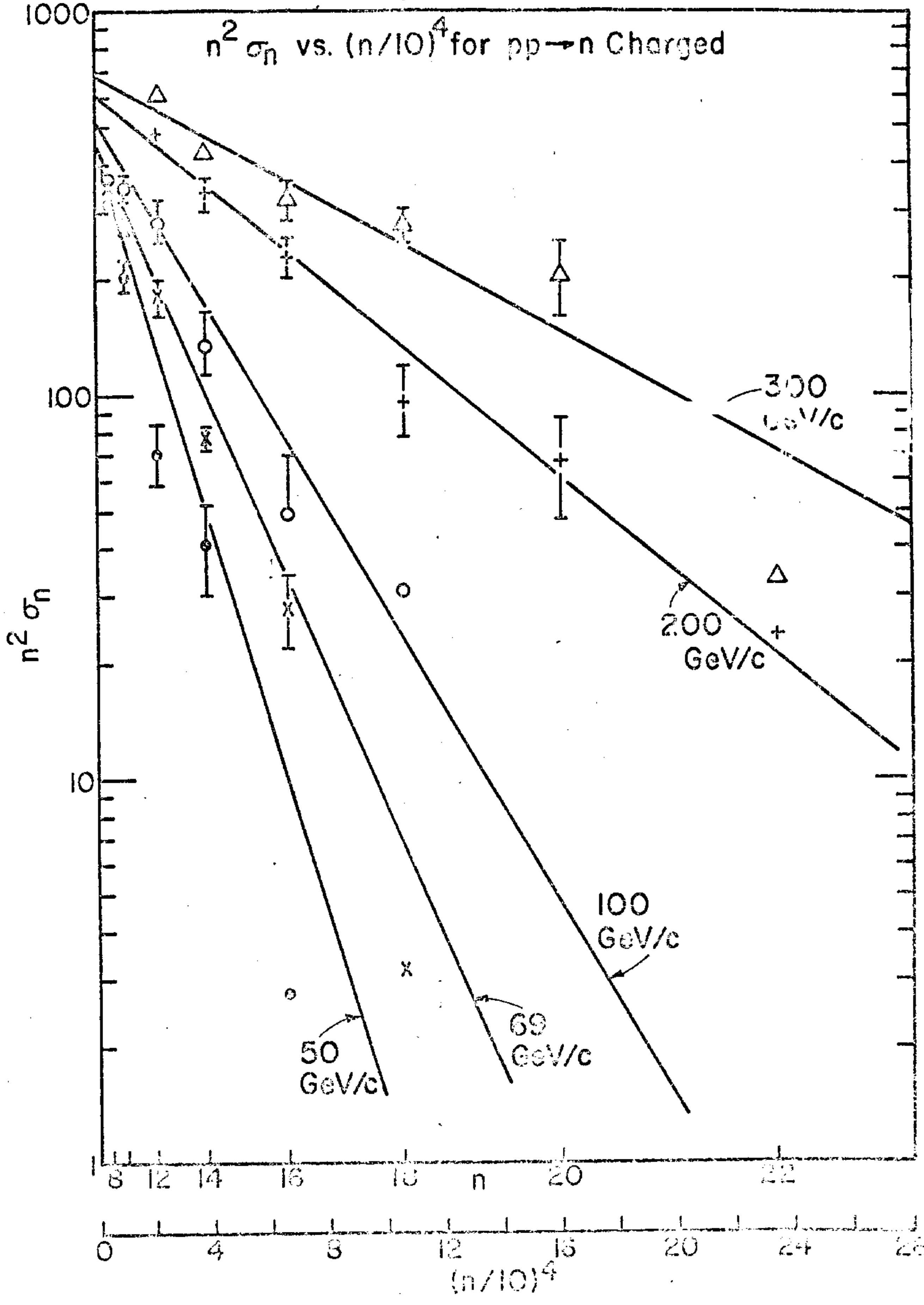
$n^2\sigma_n$ vs. $(n/10)^4$
for pp → n
Charged Particles
50 GeV/c
69 GeV/c
100 GeV/c
200 GeV/c
300 GeV/c
300 GeV/c
200 GeV/c
100 GeV/c
69 GeV/c
50 GeV/c
1000
100
10
1
$n^2\sigma_n$
8
12
14
16
18
n
20
22
0
4
8
12
16
20
24
28
$(n/10)^4$

Figure 2



$n^2 \sigma_n$ vs. $(n/10)^4$ for $\pi^- p \rightarrow$ n Charged at 50 GeV/c
$\pi^- p \rightarrow$ n Charged at 33.8 GeV/c
$n^2 \sigma_n$
1000
100
10
1
50 GeV/c
$\pi^-$
33.8 GeV/c
$K^-$
8
12
14
16
18
n
20
22
0
4
8
12
16
20
24
28
$(n/10)^4$

Figure 3



$n^2 \sigma_n$ vs. $(n/10)^4$ for pp → n Charged
1000
100
10
$n^2 \sigma_n$
300 GeV/c
200 GeV/c
100 GeV/c
69 GeV/c
50 GeV/c
8
12
14
16
18
n
20
22
0
4
8
12
16
20
24
28
$(n/10)^4$